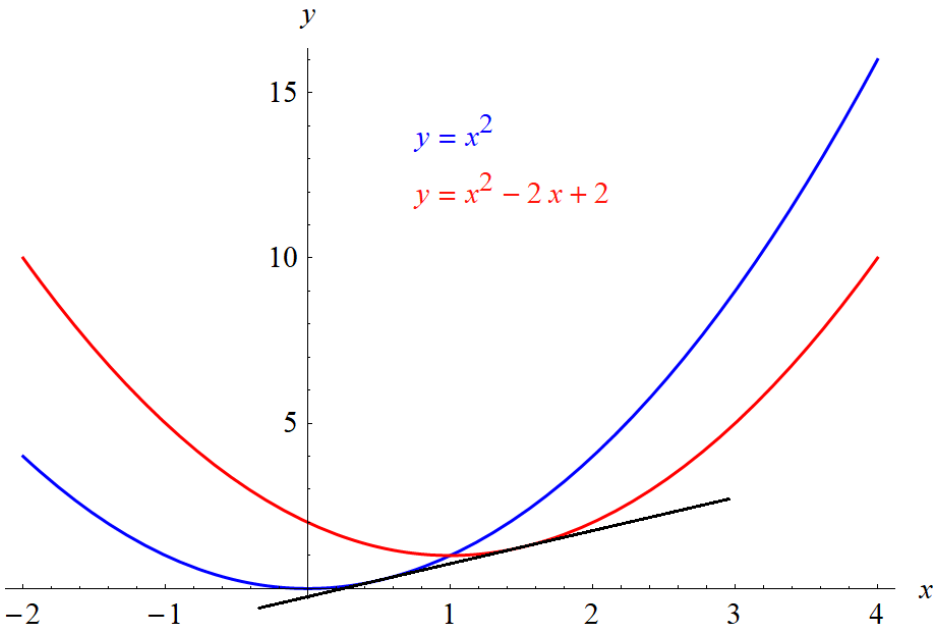
Exercise 86

Sketch the parabolas $y = x^2$ and $y = x^2 - 2x + 2$? Do you think there is a line that is tangent to both curves? If so, find its equation. If not, why not?

Solution

Below is a graph of the two parabolas along with a line that is tangent to both.



Let $P_1 = (x_1, y_1)$ be the point on $y = x^2$ that the tangent line goes through, and let $P_2 = (x_2, y_2)$ be the point on $y = x^2 - 2x + 2$ that the tangent line goes through. The slope is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(x_2^2 - 2x_2 + 2) - (x_1^2)}{x_2 - x_1}. \tag{1}$$

The derivative of the first parabola is

$$y' = 2x.$$

At $x = x_1$, the slope must be m.

$$2x_1 = m \tag{2}$$

The derivative of the second parabola is

$$y' = 2x - 2.$$

At $x = x_2$, the slope must be m.

$$2x_2 - 2 = m \tag{3}$$

Solve equations (1), (2), and (3) for x_1, x_2, and m.

$$x_1 = \frac{1}{2} \qquad x_2 = \frac{3}{2} \qquad m = 1$$

The point P_1 is $\left(\frac{1}{2}, \frac{1}{4}\right)$. Therefore, the line tangent to both parabolas is

$$y - \frac{1}{4} = 1\left(x - \frac{1}{2}\right),$$

or

$$y = x - \frac{1}{4}.$$